

Electrically controllable RKKY interaction in semiconductor quantum wires

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We demonstrate in theory that it is possible to all-electrically manipulate the RKKY interaction in a quasi-one-dimensional electron gas embedded in a semiconductor heterostructure, in the presence of Rashba and Dresselhaus spin-orbit interaction. In an undoped semiconductor quantum wire where intermediate excitations are gapped, the interaction becomes the short-ranged Bloembergen-Rowland superexchange interaction. Owing to the interplay of different types of spin-orbit interaction, the interaction can be controlled to realize various spin models, e.g., isotropic and anisotropic Heisenberg-like models, Ising-like models with additional Dzyaloshinsky-Moriya terms, by tuning the external electric field and designing the crystallographic directions. Such controllable interaction forms a basis for quantum computing with localized spins and quantum matters in spin lattices.

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All-electrical quantum manipulation of the spin degree of freedom of electrons and/or magnetic ions in semiconductors is a central issue in the fields of spintronics and quantum information processing.^{1–6} Electron spins in semiconductors have long coherence time and cost very low energy to flip.⁷ These features have obvious advantages for solid-state quantum information processing where spins of electrons and magnetic ions have been proposed as a candidate of qubits.⁸ The Heisenberg-like exchange interaction between two electrons confined in neighboring quantum dots can be controlled electrically via changing the wave function overlap of the two electrons.⁹ The distance between the neighboring dots is crucial for the strength of the exchange interaction. In order to achieve lower accuracy thresholds for quantum error correction, the implementation of coherent long-distance interaction between two qubits is desirable. Optical field can provide a practical way to realize the remote coupling between two local spins via exchange interaction mediated by the cavity modes,¹⁰ and optically generated excitons and/or electrons.^{11,12} Optical control can be realized in femtosecond processes and made robust against decoherence. The limitation of the spot size of laser beams, however, hinders the integration of qubits under optical control. It is therefore legitimate to design quantum gates based on electrically tunable remote coupling between two spins.

The RKKY interaction is an indirect exchange interaction between localized spins mediated by itinerant electrons in semiconductors or metals.^{13–16} The local spins can be magnetic ions or electron spins in quantum dots.^{17–20} A particularly interesting system is quantum dots doped with single magnetic ions which have strong s - d exchange interaction with itinerant electrons.¹ Since the RKKY interaction is mediated by itinerant electrons, the effects of spin-orbit interaction (SOI) are inevitable in conventional zinc-blende semiconductors due to breaking of the crystal inversion symmetry, i.e., Dresselhaus SOI (DSOI), and the structural symmetry, i.e., Rashba SOI (RSOI).^{21–23} The SOI is one of the major sources of spin decoherence and leads to an anisotropy in the relevant exchange interaction.^{14,24} Such anisotropy in the RKKY interaction arising from the SOIs is a

resource to be exploited in this paper for electrical control of various types of spin interactions, which is not available in systems without SOIs.

In this work, we wish to draw attention to the possibility of creating spin chains or lattices on semiconductor heterostructures. State-of-the-art e-beam lithography makes it possible to fabricate such structures. We demonstrate theoretically that the SOIs can be used to manipulate electrically the symmetry type of the spin-spin interaction. The analytical expression of the RKKY interaction shows the possibility of implementing different quantum spin models by changing the strengths of the RSOI and DSOI, e.g., isotropic and anisotropic Heisenberg models and Ising-like model. A man-made spin lattice or chain mediated by this spin-spin interaction would exhibit rich quantum phases.

First we consider two local spins \mathbf{S}_1 and \mathbf{S}_2 located at R_1 and R_2 , mediated by electrons occupying the lowest subband of a quantum wire in the presence of both the RSOI and DSOI [see Fig. 1(a)]. The Hamiltonian of the system contains the single-particle part H_0 and the s - d exchange interaction H_1 as

$$H = H_0 + H_1, \quad (1a)$$

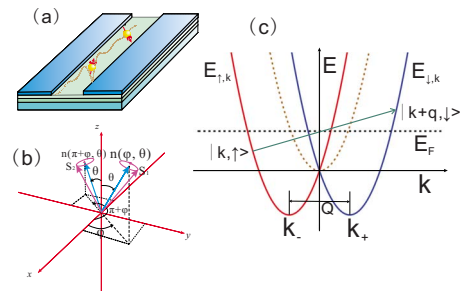


FIG. 1. (Color online) (a) Schematic of two local spins in a quantum wire, with electrically controllable RKKY interaction in the presence of RSOI and DSOI. (b) Schematic of the angles θ and φ in the twisted RKKY interaction. (c) Spin-split electron bands in the presence of the SOI. The dashed curve denotes the band without the SOI. The arrow indicates the spin-flip scattering.

TABLE I. The angles θ and φ describing the effect of RSOI and DSOI in different crystallographic planes.

Crystallographic planes	H_0	φ	θ
(001)	$H_0^{(001)} = \frac{\hbar^2 k^2}{2m^*} + \alpha(\mathbf{k} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma} + \beta(\mathbf{k} \cdot \hat{\mathbf{x}})(\boldsymbol{\sigma} \cdot \hat{\mathbf{x}})$	$\arctan(\frac{\beta}{\alpha})$	$\frac{m^*}{\hbar^2} \sqrt{\alpha^2 + \beta^2} R_{12} $
(110)	$H_0^{(110)} = \frac{\hbar^2 k^2}{2m^*} + \alpha(\mathbf{k} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma} - \frac{\beta}{2}(\mathbf{k} \cdot \hat{\mathbf{x}})(\boldsymbol{\sigma} \cdot \hat{\mathbf{z}})$	$\pi - \arctan(\frac{\beta}{2\alpha})$	$\frac{m^*}{\hbar^2} \sqrt{\alpha^2 + \frac{\beta^2}{4}} R_{12} $
(111)	$H_0^{(111)} = \frac{\hbar^2 k^2}{2m^*} + (\alpha - \frac{2}{\sqrt{3}}\beta)(\mathbf{k} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma}$	0	$\frac{m^*}{\hbar^2} (\frac{2}{\sqrt{3}}\beta - \alpha) R_{12} $

$$H_0 = \sum_{k, \eta} E_{k\eta} c_{k\eta}^\dagger c_{k, \eta}, \quad (1b)$$

$$H_1 = J \sum_{k, q, i, \eta, \eta'} e^{-iqR_i} c_{k+q, \eta'}^\dagger c_{k, \eta} \boldsymbol{\sigma}_{\eta' \eta} \cdot \mathbf{S}_i, \quad (1c)$$

where $c_{k\eta}$ annihilates an electron with quasimomentum k and spin η , $\boldsymbol{\sigma}$ denotes the Pauli matrices, and J is the strength of the s - d exchange interaction between itinerant electrons and the local spins.

The noninteracting electron energy $E_{k\eta}$ determined by the single-particle Hamiltonian $H_0 = \hbar^2 k^2 / 2m^* + V(y) + H_{\text{SO}}$, is spin-dependent due to the SOI. Above $V(y)$ is the transverse confining potential along the y axis for electrons in the heterostructure, m^* is the electron effective mass, and $H_{\text{SO}} = \mathbf{B}_{\text{eff}}(k) \cdot \boldsymbol{\sigma}$ is the SOI which is equivalent to a momentum-dependent effective magnetic field $\mathbf{B}_{\text{eff}}(k)$. The direction of the effective magnetic field depends on the crystallographic plane and its strength is proportional to the quasimomentum in the linear SOI regime. For example, for typical crystallographic planes (001), (110), and (111), the effective field $\mathbf{B}_{\text{eff}} = k(\beta, -\alpha, 0)$, $k(0, -\alpha, -\beta/2)$, and $k(0, -\alpha + 2\beta/\sqrt{3}, 0)$ in turn (see Table I), where α and β are the strengths of the RSOI and DSOI, respectively. If we choose the effective magnetic field as the quantization direction z' for the spin, the energy bands are split into two as [see Fig. 1(c)]

$$E_{k, \pm} = \frac{\hbar^2}{2m^*} \left(k \pm \frac{Q}{2} \right)^2, \quad (2)$$

with the minima shifted in the momentum space by

$$\pm Q/2 \equiv (m^*/\hbar^2) B_{\text{eff}}(k)/k, \quad (3)$$

for the spin parallel or antiparallel to the z' axis, respectively.

The RKKY interaction can be obtained using the Keldysh Green's function²⁵

$$H_{1,2}^{\text{RKKY}} = -\frac{1}{\pi} \text{Im} \int_{-\infty}^{\varepsilon_F} J^2 d\varepsilon \text{Tr}[(\mathbf{S}_1 \cdot \boldsymbol{\sigma}) G(R_{12}; \varepsilon + i0^+) \times (\mathbf{S}_2 \cdot \boldsymbol{\sigma}) G(-\mathbf{R}_{12}; \varepsilon + i0^+)], \quad (4)$$

where ε_F is the Fermi energy measured from the energy minimum of the electron bands including the SOI, $R_{12} \equiv R_1 - R_2$, and Tr means a partial trace over the spins of itinerant electrons. With the spin quantization direction z' chosen along the effective magnetic field \mathbf{B}_{eff} , the RKKY interaction has the form

$$H_{1,2}^{\text{RKKY}} = F_1(q_F | R_{12}|) \left(S_1^z S_2^z + \frac{1}{2} S_1^+ S_2^- e^{i2\theta} + \frac{1}{2} S_1^- S_2^+ e^{-i2\theta} \right), \quad (5)$$

where $q_F = \sqrt{2m^* \varepsilon_F / \hbar^2}$ is the Fermi wave vector of each spin-split band measured from the band minimum, the phase angle $\theta \equiv Q|R_{12}|/2$, and the range function is

$$F_1(q_F | R_{12}|) = \frac{4J^2 m^*}{\pi \hbar^2} \left[\text{Si}(2q_F | R_{12}|) - \frac{\pi}{2} \right], \quad (6)$$

with $\text{Si}(x)$ being the sine integral function. The physical processes underlying different terms in the RKKY interaction is clearly identified: the first term in the bracket of Eq. (5) arises from the spin-conserving scattering within each spin-split band and hence has exactly the same form as in systems without the SOI.²⁶ The second and third terms correspond to the spin-flip scattering between different spin-split bands and the phase factor $e^{\pm i2\theta} = e^{\pm iQR_{12}}$ is the phase shift accumulated over R_{12} by the extra momentum quantization Q that separates the minima of the two bands. The RKKY interaction in Eq. (5) is general for arbitrary crystallographic planes and quantum wire orientations.

Back into the laboratory coordinate systems, the RKKY interaction is transformed to

$$H_{1,2}^{\text{RKKY}} = F_1(q_F | R_{12}|) \{ \mathbf{S}_1 \cdot \mathbf{S}_2 - 2S_{1z} S_{2z} + 2[\mathbf{S}_1 \cdot \mathbf{n}(\pi + \varphi, \theta)] \times [\mathbf{S}_2 \cdot \mathbf{n}(\varphi, \theta)] \}, \quad (7)$$

where $\mathbf{n}(\varphi, \theta) = (\cos \varphi \sin \theta, \sin \varphi \sin \theta, \cos \theta)$. The angle θ and angle φ are given in Table I depend on the crystallographic planes where the quantum wire is embedded. The angle θ describes how both the RSOI and DSOI twist the two local spins away from the z axis and the angle φ determines the in-plane twist of the spin orientation [see Fig. 1(b)]. If the Dresselhaus interaction is absent, our result is reduced to the previous work.¹⁴

The system with both the DSOI and RSOI has a great extent of controllability, owing to the interplay of the two types of SOIs and the sensitive dependence of DSOI on the crystallographic plane and the quantum wire orientation. Equation (7) works for all but the (110) crystallographic plane. For a quantum wire embedded in the heterostructure grown along the $[110]$ direction, the RKKY interaction becomes $H(\varepsilon_F)^{\text{RKKY}} = F_1(q_F | R_{12}|) \{ \mathbf{S}_1 \cdot \mathbf{S}_2 - 2S_{1x} S_{2x} + 2[\mathbf{S}_1 \cdot \mathbf{n}(\pi + \varphi, \theta)] [\mathbf{S}_2 \cdot \mathbf{n}(\varphi, \theta)] \}$.²⁷ The RKKY interaction shows isotropic behavior at $\alpha = 2\beta/\sqrt{3}$ when the quantum wire is embedded on the (111) crystallographic plane, because the RSOI and DSOI have the same dependence on the in-plane

TABLE II. Examples of various quantum spin models. \pm corresponds to the condition that the quantum wire is embedded in the (001) plane but along the [100] (+) or the [010] (−) direction, respectively.

Crystallographic planes	Spin models	$H(\varepsilon_F)^{RKKY}$	θ	φ
(001)	Heisenberg model	$F_1 \mathbf{S}_1 \cdot \mathbf{S}_2$	$k\pi, (k \in \mathbb{Z})$	Arbitrary values
	Anisotropic Heisenberg model	$F_1[S_{1x}S_{2x} - S_{1y}S_{2y} - S_{1z}S_{2z}]$	$\pi/2$	$\pi/2$
	Ising model with a DM term	$F_1[S_{1x}S_{2x} \pm (\mathbf{S}_1 \times \mathbf{S}_2)_x]$	$\pi/4$	$\pi/2$
(110)	Heisenberg model	$F_1 \mathbf{S}_1 \cdot \mathbf{S}_2$	$k\pi, (k \in \mathbb{Z})$	Arbitrary values
	Anisotropic Heisenberg model	$F_1[-S_{1x}S_{2x} - S_{1y}S_{2y} + S_{1z}S_{2z}]$	$\pi/2$	$\pi/2$
	Ising model with a DM term	$F_1[S_{1z}S_{2z} + (\mathbf{S}_1 \times \mathbf{S}_2)_z]$	$\pi/4$	$\pi/2$
(111)	Heisenberg model	$F_1 \mathbf{S}_1 \cdot \mathbf{S}_2$	$k\pi, (k \in \mathbb{Z})$	0
	Anisotropic Heisenberg model	$F_1[-S_{1x}S_{2x} + S_{1y}S_{2y} - S_{1z}S_{2z}]$	$\pi/2$	0
	Ising model with a DM term	$F_1[S_{1y}S_{2y} + (\mathbf{S}_1 \times \mathbf{S}_2)_y]$	$\pi/4$	0

momentum (see Table I). It means it is possible to switch on/off the SOIs.²⁸ When the quantum wire is embedded in arbitrary crystallographic directions on (001) plane, i.e., in the direction with an angle ϑ respect to the [100] direction, the formalism of RKKY interaction remains the same and the angle θ and angle φ should be redefined as $\theta \equiv m^* \sqrt{\alpha^2 + \beta^2 + 2\alpha\beta \sin 2\vartheta} |R_{12}| / \hbar^2$, $\varphi \equiv \arctan[\beta \cos 2\vartheta / (\alpha + \beta \sin 2\vartheta)]$. Note that when the quantum wire is embedded along the $\bar{1}10$ direction on the (001) plane, $\theta \propto |\alpha - \beta|$ and $\varphi = 0$, it would give us another way to switch on/off the SOIs by tuning the strength of the external electric field.

The most important difference between our result and the previous works^{11,12} is that the interplay between the RSOI and DSOI offers us a new way to control the spin-spin interaction. Due to the interplay of the DSOI and RSOI, the RKKY interaction presents, in addition to the usual Heisenberg-like exchange term, not only an Ising-type anisotropic term, but also a twisted Dzyaloshinsky-Moriya (DM)-like term which twists the local spins [see the third term in Eq. (7)].

Tuning the parameters θ and φ , we can rotate the local spins in a spin space [see Fig. 1(b)], and construct various kinds of quantum spin models. From Eq. (7), we can realize many spin models by tuning the strengths of the SOIs (see Table II), for instance, isotropic Heisenberg-like model $F_1(q_F |R_{12}|) \mathbf{S}_1 \cdot \mathbf{S}_2$, the Ising-like model with an additional DM-like interaction term $H(\varepsilon_F)^{RKKY} = F_1(q_F |R_{12}|) [S_{1x}S_{2x} \pm (\mathbf{S}_1 \times \mathbf{S}_2)_x]$ and the anisotropic Heisenberg model $H(\varepsilon_F)^{RKKY} = F_1(|R_{12}|) [S_{1x}S_{2x} - S_{1y}S_{2y} - S_{1z}S_{2z}]$. In the long-range anisotropic Heisenberg model, along the in-plane and the out-of-plane directions, the spin correlations, being ferromagnetic or antiferromagnetic, are different.

The conditions mentioned in Table II, e.g., $\theta = k\pi$, are realizable in a narrow band gap semiconductor InSb quantum well with 10 nm thickness at a specific perpendicular electric field $E \approx 50$ kV/cm. The SOI is strong in narrow band gap semiconductor quantum wells, e.g., HgCdTe QWs, in which α ranges from 10^{-13} eV m to 10^{-10} eV m depending on the external gate voltage, thickness of QW, and electron density.²⁹ Choosing a proper external electric field, one can realize the switching between different spin models.

All-electrical two-qubit gates can be implemented with

the RKKY interaction, being either the Heisenberg-like interaction or the Ising-like interaction. The controllability of the interaction symmetry in the SOI systems gives us further flexibility of realizing various types of two-qubit gates such as the $\sqrt{\text{SWAP}}$ gate and the controlled-phase gate, either of which, plus one-spin operations, constitute a complete set of gates for universal quantum computing. In particular, the isotropic Heisenberg interaction can be used for both one-qubit and two-qubit gates.³⁰ For an estimation of the operation rate, we notice that the exchange coupling J can be tuned to 1 meV by external electric fields, which indicates that two-qubit gates with a picosecond cycle would be possible if the electrical control can be done at that rate.

This approach of constructing electrical-controllable spin-spin interaction outlined above can be extended to more complicated structures. Here we propose that a single pair of spin qubits be replaced with an array of local spins, i.e., a spin lattice or chain, which is defined on quantum wires embedded in semiconductor heterostructures. Spin lattices are platforms of a wealth of many-body physics and quantum phenomena such as quantum phase transitions and may also be a computing resource such as in quantum simulation of condensed matter systems. The RKKY interaction is a long-ranged interaction since its asymptotic behavior $\lim_{R_{12} \rightarrow \infty} F_1(q_F R_{12}) \rightarrow \cos(2q_F R_{12}) / R_{12}$ that is inversely linear in distance R_{12} with an oscillation superimposed. In practice, precise positioning of spins for realizing an artificial spin lattice proposed here is still a great challenge. The long-range interaction would make the quantum physics richer and more complicated. Such systems often manifest quantum phase transitions governed by parameters such as the external field and concentration of impurities. In order to realize a short-ranged spin-spin interaction, we could use a one-dimensional intrinsic narrow band gap semiconductor quantum wire in which the virtual excitations between the valence and conduction bands, in lieu of itinerant electrons in doped semiconductors, mediate the interaction. The range function of the spin-spin interaction becomes $F_1(q_F R) \propto e^{-\lambda R}$ from Eq. (1a) utilizing the Keldysh Green's function. The interaction length $\lambda \approx \hbar / \sqrt{2m^* \Delta}$, mostly determined by the electron effective mass considering the large mass of the holes, can be tuned from 10 nm to infinity by adjusting the band gap Δ of, e.g., a HgCdTe quantum well from 0.1 eV to zero where a

quantum phase transition takes place.³¹ It provides us a new way to control the range of the spin-spin interaction.³² Using virtual excitations to mediate the spin-spin interaction also largely avoids the fast optical decoherence.¹¹ In the spin lattice, one can also control the spin-spin interaction spatially, and realize different spin models in a spin lattice electrically, i.e., anisotropic Heisenberg model and Ising-like model with an additional DM term. The DM-like term can induce the interesting spiral phase in the spin chain in which the spins rotate along the $\mathbf{S}_i \times \mathbf{S}_{i+1}$ axis. Using the twisted DM term induced by the SOIs, one can use an electric field pulse, which propagates along the spin chain, to generate a propagating spin wave along the spin chain, and this spin wave excitation is actually a low power consumption spin current since one only needs to flip the neighboring spins without drifting of electrons.

However, the SOI in semiconductor low-dimensional electron gases is a double-edged sword, since the spin relaxation is typically dominated by the D'yakonov-Perel' (DP) mechanism,³³ and is enhanced with increasing the SOI. The spin decoherence induced by the SOIs is strongly suppressed in this spin lattice due to the quasi-one-dimensional geometry of quantum wires, since only one single point in the k -space satisfies the momentum and energy conservation conditions for real excitations.

In summary, we propose all-electrical manipulation of the spin-spin interaction via the RSOI and DSOI of electrons localized in quantum wires. This RKKY interaction can be both controlled in magnitude and symmetry-tuned heavily by adjusting the strength of SOIs, Fermi energy and crystallographic planes, and display different types of spin-spin interactions. Both isotropic and anisotropic Heisenberg models and Ising-like models with additional DM terms could be realized. The anisotropy and twisted term in the RKKY interaction caused by the SOIs can be removed by adjusting the strength of SOIs. The parameters related to constructing the spin models can be electrically controlled. Such *in situ* controllability may be used for observing quantum phase transitions in spin lattices without external magnetic fields. The short-ranged spin-spin interaction can be realized utilizing the virtual interband excitations in narrow band gap semiconductors.

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